How are Flow Conditions in Volcanic Conduits Estimated?

How can variations in magma density and velocity be estimated as a function of depth in conduits?

Using simplified equations of state to model steady-state Plinian eruptions

Core Quantitative Issue
Conservation of mass and momentum

Supporting Quantitative Issues
Equilibrium and disequilibrium
Henry's Law

SSAC - Physical Volcanology Collection
Chuck Connor – University of South Florida, Tampa

 Edited by Judy Harden 10/29/07

Preview

This module presents a calculation of the ascent velocity of magma in a volcanic conduit.

Slides 3-9 *give some background on processes operating in volcano conduits during Plinian eruptions.*

Slide 10 *states the problem. What is the ascent velocity of magma as a function of depth?*

Slides 11-14 *analyze the problem and prompt you to design a plan to solve it. The problem breaks down into parts: estimating the solubility of volatiles in the magma, the magma density, and the velocity.*

Slide 15 *illustrates a spreadsheet that calculates an answer.*

Slide 16 *discusses the point of the module and provides a broader volcanological context.*

Slide 17 *consists of some questions that constitute your homework assignment.*

Slide 18 *discusses the speed of sound in magma.*

Slide 19 *provides data needed to solve the end-of-module assignments.*

Steady-State Plinian Eruptions

Plinian eruptions are among the most intense volcanic eruptions on Earth, characterized by large mass of erupted pyroclasts (e.g., greater than 10^{10} kg) and high eruption column heights (e.g., greater than 20 km).

These eruptions are often characterized by apparently steady – or nearly steady – eruption conditions. It is as if a fire hose of magma flows from the vent for a period of time, driven by pressure in the magmatic reservoir that only diminishes after a significant volume of magma has erupted.

A plinian eruption column at Lascar volcano, Chile, in 1993.

Steady-State Plinian Eruptions

The volcano conduit can be represented by several flow regimes and transitions during steady-state Plinian eruptions. These include deep magma ascent due to pressure-driven flow (e.g., Poiseuille flow), bubble nucleation and bubbly flow, perhaps accompanied by crystal growth, fragmentation, and dispersed flow. These flow regimes and transitions are simplified models of the actual processes but do capture the major characteristics of the volcano conduit during steady-state Plinian eruptions.

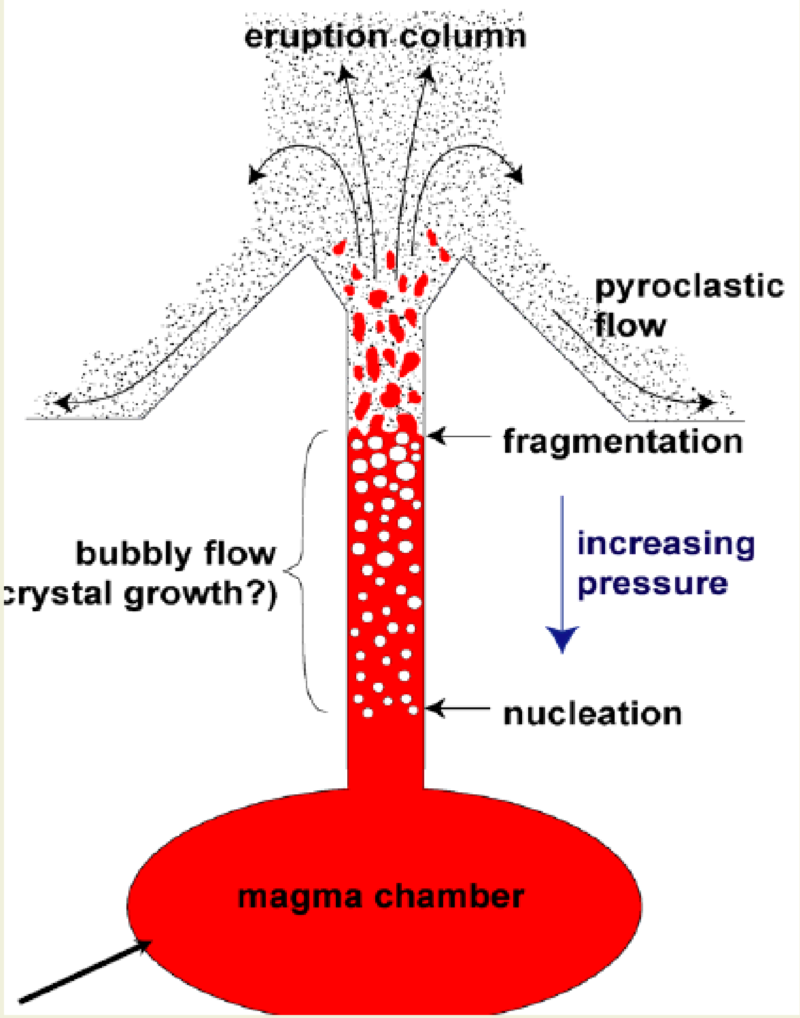


Diagram from Ed Llewellin.

[More about flow conditions at the vent]

Excess pressure initiates flow.

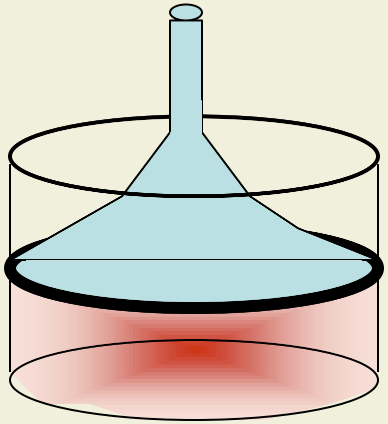
Background

Henry's Law

Henry's Law describes the relationship between mass fraction of volatiles that may be dissolved in a magma and pressure:

$$n_s = sP^{\beta}$$

where n_s is the mass fraction of volatiles in solution (dissolved in the magma), P is the pressure, s and β are constants that depend on composition of the magma. For rhyolite magmas, $\beta = 0.5$, $s = 4.1 \times 10^{-6}\ Pa^{-0.5}$; and for basalt magmas, $\beta = 0.7$ and $s = 6.8 \times 10^{10}\ Pa^{-0.7}$.



Henry's Law for magmas: The higher the pressure, the more volatiles can be dissolved in the magma.

Please check your understanding of Henry's Law. What is the solubility of water in a rhyolite magma at a depth of 1 km, assuming hydrostatic pressure, $\rho_{magma} = 2500\ kg\ m^{-3}$? Recall $P = \rho gh$. Your answer should be approximately 0.02. How much can dissolve in the same magma at 250 m depth?

Henry's Law

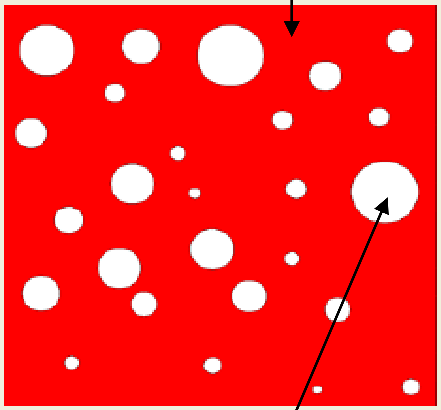
Henry's law provides guidance about the mass fraction of volatiles dissolved in the melt and the mass of volatiles exsolved from the melt – that is, the fraction found in bubbles.

The mass fraction of exsolved volatiles in the magma at a given pressure is:

$$n_{ex} = n_o - sP^{\beta}$$

where n_o is the initial mass fraction of volatiles dissolved in the melt before any bubble formation takes place.

Mass fraction remaining in melt (estimated with Henry's Law)



Mass fraction exsolved (total volatiles in the bubbles)

Initial volatile fraction = mass fraction in melt + mass fraction in bubbles

Henry’s Law is not enough.

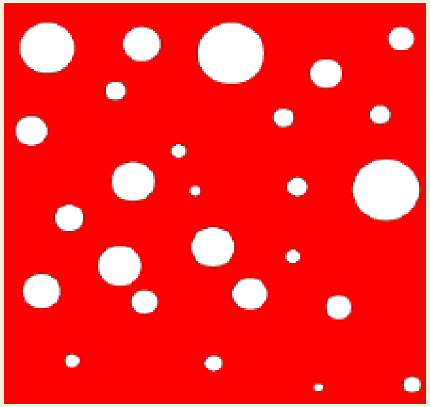
Henry’s Law is a nice leading order indicator of saturation, but in reality there is more to bubble growth: nucleation, diffusion, and decompression.

Henry’s Law applies to equilibrium conditions in a homogeneous magma. Ascending magmas, however, have little respect for equilibrium conditions. It takes time and energy for bubbles to form – a process called bubble nucleation, and it takes time for them to grow. Such factors are not considered in Henry’s Law.

When opening a bottle of beer the pressure changes instantly, changing the saturation of volatiles, which come out of solution. Long after the pressure changes, however, bubbles continue to nucleate and rise through the beer, a situation not well-predicted by Henry’s Law.

What is homogeneous bubble nucleation?

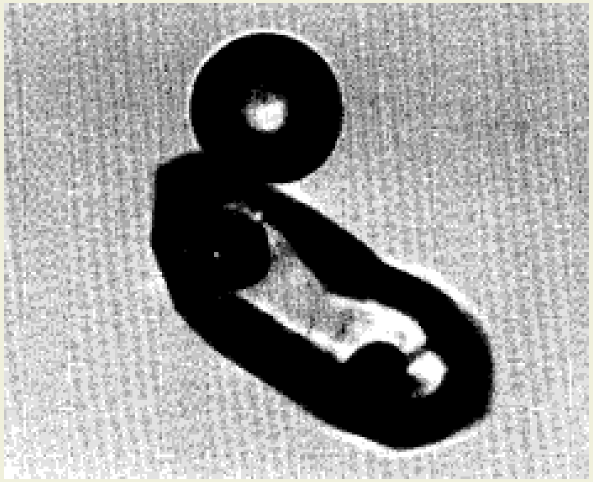
Bubbles form when the magma becomes super-saturated in volatiles. Henry's law provides a clue about these conditions, but experiments indicate that, at least in some conditions, magmas become extremely super-saturated before bubbles actually nucleate and begin to grow. In pure melts, super-saturation may be of order ΔP=80 – 100 MPa before bubble nucleation proceeds rapidly.



Bubbles nucleating in a "homogeneous" melt, that is free of crystals, xenoliths, and related material – an example of homogeneous nucleation.

What is heterogeneous bubble nucleation?

In reality, super-saturation pressures can be reached before bubbles ever form. In such magmas, bubble growth often occurs when bubbles nucleate on crystal surfaces. In pure melts, super-saturation may be of order ΔP= 5 to 30 MPa, less than the overpressures seen in homogenous nucleation experiments.



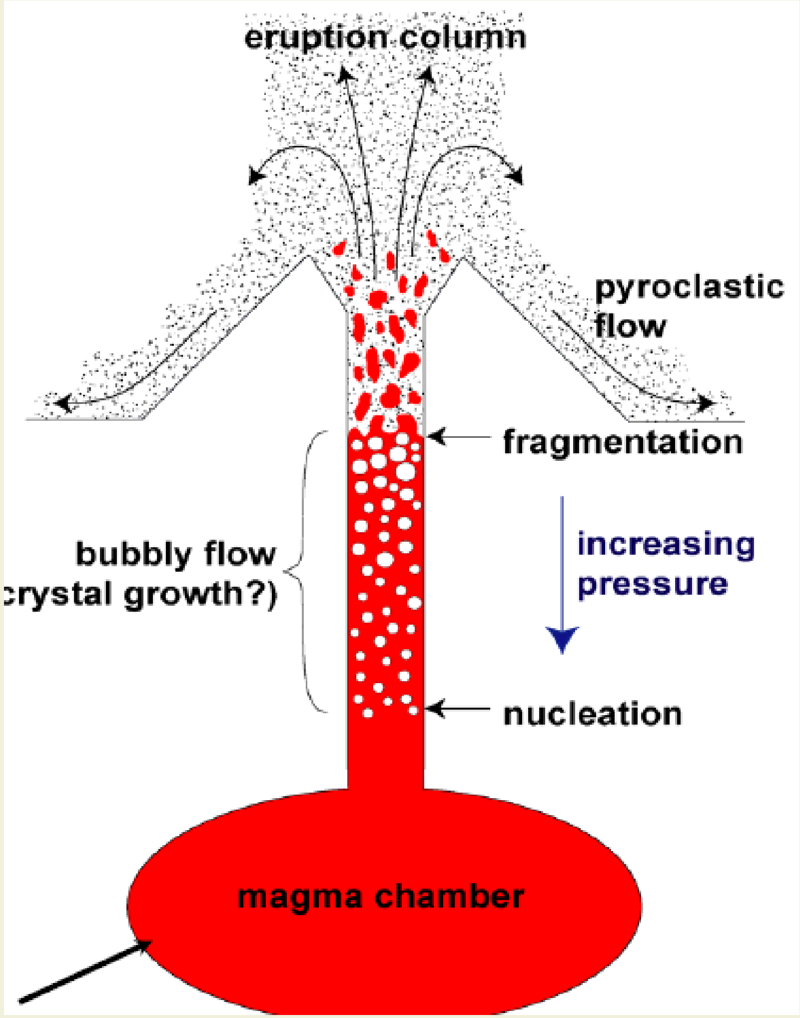
A bubble residing at its nucleation site on a zircon crystal. The mineral grain creates a site for nucleation of the bubble. The field of view is about 500 μm. (Photo from experiments conducted by Hurwitz, S., and O. Navon, 1994, Bubble nucleation in rhyolite melts: Experiments at high pressure, temperature and water content, Earth and Planetary Science Letters, 122: 267-280).

Problem

Given pressure conditions in a volcano conduit as a function of depth, what is the ascent velocity of the magma?

Use Henry's law and conservation of mass to estimate ascent velocity.

It is necessary to make some simplifying assumptions in order to estimate flow in the conduit. We will solve the problem in simplified form, by assuming one dimensional steady-state flow. We will assume that the pressure conditions along the conduit are known, and solve for velocity from pressure, using Henry's Law. More general solutions exist, to actually solve for the pressure as well, but these require more elaborate computer programs.



Excess pressure initiates flow.

Diagram from Ed Llewellin.

Designing a Plan, Part 1

Given pressure conditions in a volcano conduit as a function of depth, what is the ascent velocity of the magma?

Give answer in meters per second.

You will need to:

- **Calculate the solubility of volatiles in magma as a function of depth with Henry's law.**
- **Calculate the density of the ascending magma as a function of depth.**
- **Calculate the average ascent velocity as a function of depth in the conduit, using conservation of mass.**

***Notes*:**

(1) Assume that Henry's law applies and ignore super-saturation for the time being.

(2) Also, assume that no gas is lost from the conduit – that is, all of the exsolved gas is retained in bubbles. Of course this is an important assumption, as some gas could be lost through the walls of the conduit.

(3) As you develop a plan for solving this problem, keep careful track of the additional assumptions that you are required to make in order to solve the problem!

Designing a Plan, Part 2

Calculate the solubility of volatiles in magma with Henry's law.

As a first step, estimate the solubility of water in the magma and the mass fraction of water exsolved from the magma.

Henry's law gives the solubility as a function of pressure, assuming that magma is in equilibrium (not super-saturated). The mass fraction of exsolved volatiles can be calculated if the initial volatile content in the magma is known.

$$n_s = 4.1 \times 10^{-6} \sqrt{P}$$

$$n_{ex} = \frac{n_o - n_s}{1 - n_s}$$

where n_s is the solubility of water in rhyolite magma, P is the pressure in Pa, n_{ex} is the mass fraction of volatiles exsolved from the magma, and n_o is the initial volatile content in the magma.

This representation of Henry's Law is called an equation of state. In the context of this problem, this means that the equation describes the behavior of one variable (solubility of water or mass fraction of exsolved volatiles) in terms of another physical property of the system (pressure). This particular equation of state is only valid for rhyolite magmas and water. The coefficients of the equation change with magma composition and volatile species. Of course, use of this equation of state assumes that Henry's Law is valid. Many models used in the Earth Sciences depend on equations of state and it is important to always consider their validity when using a model to solve a specific problem.

Designing a Plan, Part 2

Calculate the density of the ascending magma.

The density of the magma (in this case, the mixture of melt and bubbles) can be estimated with the ideal gas law.

The density of the mixture under specific pressure conditions, and assuming that Henry's Law applies, is given by:

$$\frac{1}{\rho} = \frac{n_{ex}RT}{P} + \frac{1-n_{ex}}{\sigma}$$

where ρ is the bulk density of the magma-gas mixture, n_{ex} is the mass fraction of exsolved volatiles in magma in unsaturated conditions, R is the gas constant expressed in terms of mass (462 J kg^{-1} K^{-1}) for water vapor, T is temperature (K), P is pressure (Pa) and σ is the density of the melt in unsaturated conditions.

Check the units of this equation. Note that the gas constant for water vapor is expressed in terms of J kg^{-1} K^{-1}. What are the normal units of the gas constant. What assumption is made in expressing units in J kg^{-1} K^{-1} about the volatiles in the magma?

Designing a Plan, Part 2

Calculate the average ascent velocity in the conduit for a rhyolitic magma with an initial volatile content of 0.05 at a temperature of 1100 K and a density of 2500 kg m^{-3}. The mass eruption rate is $3x10^7$ kg s^{-1} and the conduit radius is 30 m.

Use conservation of mass to estimate the flow velocity.

Once the density of the magma as a function of depth is estimated, it is possible to estimate the velocity of the flow along the conduit, for a given mass flow. In this case, a 1D velocity is calculated. That is, the velocity is assumed to not vary from the center to the edges of the conduit. The actual velocity will be higher in the center of the conduit and lower at the edge of the conduit.

$$u = \frac{Q}{\pi r^2 \rho}$$

where u is the average velocity, Q is the mass flow, r is the radius of the cylindrical conduit and ρ is the density of the magma, which varies as a function of depth.

Carrying Out the Plan: Spreadsheet to Calculate Ascent Velocity

	B	C	D	E	F	G	H
2	***Calculate the change in magma velocity as a function of depth in a conduit.***						
3							
4	***Given (for Rhyolite Magmas)***						
5	Initial Volatile Content	0.05	mass fraction				
6	Henry's Law (s)	0.0000041	rhyolite				
7	Henry's Law (β)	0.5	rhyolite				
8	Magma Temperature	1100	K				
9	Melt Density	2500	kg m^{-3}				
10	Gas Constant	462	J kg^{-1} K^{-1}				
11	Mass Eruption Rate	3.00E+07	kg s^{-1}				
12	Conduit Radius	30	m				
13							
14	***Calculate***						
15	Depth	Pressure	solubility	n_{ex}	1/rho	rho	*u*
16	(m)	(Mpa)	mass fraction	mass frac.		(kg m^{-3})	(m s^{-1})
17	-5000	128	0.04638620	0.00378958	0.000414	2418	4.3876894
18	-4950	125.26088	0.04588720	0.00431060	0.000416	2405	4.4113976
19	-4898.338711	122.44191	0.04536792	0.00485221	0.000418	2391	4.4372229
20	-4829.183572	118.66978	0.04466362	0.00558586	0.000422	2371	4.4742376
21	-4739.463552	113.76128	0.04373016	0.00655656	0.000427	2344	4.5270791
22	-4623.544634	107.37234	0.04248446	0.00784900	0.000434	2304	4.6049915

A cell containing a number that is given information

A cell containing a formula

At this point, be sure to implement this spreadsheet and check that your formulas duplicate the values shown. You will need this spreadsheet to complete the end-of-module assignment.

What you have done

You have calculated ascent velocity of magma in a conduit.

A fundamental challenge in volcanology is to estimate the conditions of flow and transport of magma as it moves from a deep magma reservoir to the surface. These flow conditions cannot be observed. Rather, evidence of flow conditions is preserved from ancient eruptions, as seen in the geologic record, and flow conditions can be inferred based on observations from historical eruptions.

In volcanology, we rely on mathematical solutions to provide additional insight into the nature of flow processes. These mathematical representations of flow depend on a physical understanding of magma systems. Here, you have used a great many simplifying assumptions to estimate flow and transport.

Useful papers that discuss magma ascent in detail:

Jaupart, C., 2000, Magma ascent at shallow levels, In: Sigurdsson et al., eds., Encyclopedia of Volcanoes, Academic Press, 237-245. (an accessible discussion).

Woods, A.W., 1995, The dynamics of explosive volcanic eruptions., Reviews in Geophysics 33: 495-530.

End-of-Module Assignments

1. Make sure you turn in a spreadsheet showing the worked example.

2. Conduit pressures as a function of depth are given for a rhyolite magma (see *slide 19*). Using given information on slide 15, calculate the change in solubility, density, and velocity as a function of depth. What is the approximate exit velocity of the magma from the conduit?

3. Plot a graph of the change in pressure as a function of depth in this conduit.

4. Plot a graph of the change in magma density as a function of depth. What is the approximate level of fragmentation in this conduit?

5. Plot a graph of the change in velocity as a function of depth. On this graph, indicate zones of bubbly flow and dispersed flow.

6. What assumptions have you made in these calculations?

More About the Exit Velocity

Calculate the speed of sound in the magma-gas mixture.

The speed of sound in the eruption is given by the following expression:

$$u_c = \sqrt{\frac{RT}{n_{ex}}}\left[n_{ex} + (1 - n_{ex})\frac{P}{\sigma RT}\right]$$

where u_c is the speed of sound in the magma-gas mixture, *R is the gas constant expressed in terms of mass (462 J kg^{-1} K^{-1}), T is temperature (K), n_{ex} is the mass fraction of exsolved volatiles, P is pressure (Pa), and σ is the density of the magma in unsaturated conditions.*

In "choked" flow conditions, the velocity of the magma at the vent is approximately equal to the speed of sound in the magma at the vent. It is possible to calculate the sound speed (also called the critical velocity) from the physical properties of the magma and flow conditions (specifically pressure).

Return to Slide 4

Excel Worksheet Containing Problem Data

This Excel worksheet contains the data you need to complete the EOM assignments – double click on the worksheet and copy-paste into Excel. The data extends beyond what is shown here; be sure to copy all 51 lines.

Depth	Pressure
-5000	128
-4950	125.2609
-4898.34	122.4419
-4829.18	118.6698
-4739.46	113.7613
-4623.54	107.3723
-4477.56	99.21395
-4309.77	89.61468
-4168.93	81.2946
-4048.12	73.88282
-3944.45	67.23418
-3855.72	61.24374
-3780.03	55.82444
-3715.75	50.90257
-3661.42	46.41558
-3615.75	42.31038
-3577.61	38.54197
-3545.98	35.07211
-3519.97	31.86832
-3498.76	28.90284
-3481.63	26.15166
-3467.93	23.59354
-3457.11	21.20865
-3448.64	18.9764
-3442.09	16.87029
-3441.37	16.60923
-3427.55	14.82984
-3389.25	14.58341
-3197.76	13.40494
-2774.62	11.0967

Return to E-O-M Assignments