

# What is the Volume of the 1992 Eruption of Cerro Negro Volcano, Nicaragua?

How well can we estimate the volumes of volcanic deposits?

*Using the 'exponential-thinning' model of tephra fallout to estimate volume and eruption magnitude*

Core Quantitative Issue  
Linear Regression

Supporting Quantitative Issues  
Exponential Model  
Volume estimates

SSAC - Physical Volcanology Collection  
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# Preview

**This module presents a calculation of the volume of the 1992 eruption of Cerro Negro volcano, Nicaragua.**

*Slide 3-6* give some background on tephra fallout and the 1992 eruption.

*Slide 7* states the problem. What is the volume of the tephra fallout deposit?

*Slides 8 and 9* develop a plan for solving the problem. An exponential thinning model is assumed to describe the change in thickness of the deposit with distance from the volcano. This model is used to estimate the volume of the deposit.

*Slides 10-13* illustrate the solution of the problem, developing a spreadsheet to calculate the volume.

*Slide 14* discusses the point of the module and provides a broader volcanological context.

*Slide 15* consists of some questions that constitute your homework assignment.

*Slides 16-20* are endnotes for elaboration and reference.

*Slides 21 and 22* contain additional information required to answer question 4 in the end of module assignments.

# Background

## What is tephra fallout?

tephra – a term that refers to all particles ejected explosively from a volcano, regardless of size, shape, or composition.

The term was introduced by the famous Icelandic volcanologist Thorarinsson to describe particles that travel through the air, are carried by wind, and eventually land on the ground.

Therefore, tephra fallout deposits (varieties called “airfall deposits”, “pyroclastic fallout deposits”) are completely comprised of tephra and have characteristics that distinguish them from other types of flow phenomena. Tephra fallout deposits mantle the terrain and thin, often exponentially, with distance from the volcano.



*Photo by C. Connor*

Under the plume of the 1992 eruption of Cerro Negro volcano, Nicaragua, during the waning stages of the eruption. More than 80 cm of tephra had already accumulated in this area when the photograph was taken, virtually destroying the forest of rich tropical growth that covered the area just a few days before.

For more about tephra terminology:

<http://volcanoes.usgs.gov/Hazards/What/Tephra/tephraterms.html>

# Background

## How “big” are volcanic eruptions?

The magnitude of explosive volcanic eruptions is sometimes classified by the volume of tephra erupted. The volcano explosivity index (VEI) (developed by Chris Newhall and Steve Self, and used by the Smithsonian Institution’s Global Volcanism Network) is linked directly to tephra volume.

	0	1	2	3	4	5	6	7	8
General Description	Non-Explosive	Small	Moderate	Moderate-Large	Large	Very Large			
Volume of Tephra (m <sup>3</sup> )	1x10 <sup>4</sup>	1x10 <sup>6</sup>	1x10 <sup>7</sup>	1x10 <sup>8</sup>	1x10 <sup>9</sup>	1x10 <sup>10</sup>	1x10 <sup>11</sup>	1x10 <sup>12</sup>	
Cloud Column Height (km) Above crater Above sea level	<0.1	0.1-1	1-5	3-15	10-25	>25			
Qualitative Description	"Gentle,"	"Effusive"	"Explosive"		"Cataclysmic," "Severe," "violent,"		"paroxysmal,"	"colossal"	
Eruption Type	Hawaiian	Strombolian		Vulcanian		Plinian	Ultra-Plinian		
Duration (continuous blast)		<1 hour		1-6 hrs		6-12 hrs		>12 hrs	
CAVW max explosivity (most explosive activity listed in CAVW)	Lava flow	Phreatic		Explosion or Nuée ardente					
Tropospheric Injection	Negligible	Minor	Moderate	Substantial					
Stratospheric Injection	None	None	None	Possible	Definite	Significant			
Eruptions (total in file)	755	963	3631	924	307	106	46	4	0



Photo by C. Connor

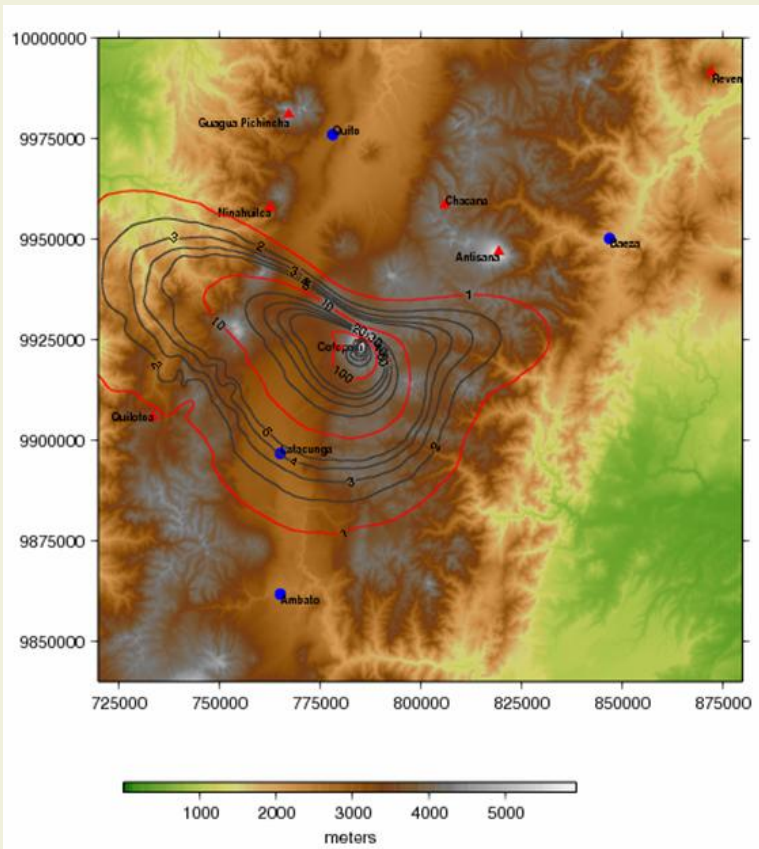
The eruption column at Cerro Negro during the waning stages of 1992 activity. Based on tephra volume, this was a VEI 3 eruption. The cinder cone itself, truncated by the eruption, is just visible at the bottom of the photo.

Learn more about the Volcano Explosivity Index:

<http://www.volcano.si.edu/world/eruptioncriteria.cfm#VEI>

# Background

## How is tephra volume estimated?



Isopach maps contour the thickness of a deposit, in this case thickness of tephra fallout. Isomass maps contour the mass per unit area of a deposit. Example from Cotopaxi volcano, Ecuador. Connor and Connor, 2006.

Several methods exist for measuring the volume of tephra fallout deposits:

(1) A simple method involves interpolating isopachs from field observations, then finding the area of each isopach, and hence the volume. While simple, this method tends to underestimate volume because it does not account for tephra that is widely dispersed – especially beyond the limit of the isopachs on the map!

(2) A better method uses information about the eruptive process. As deposits often thin exponentially with distance from the volcano, this fact can be used to estimate volume. You will use this method. Similar methods use “power-law thinning”, rather than “exponential thinning” of the deposit.

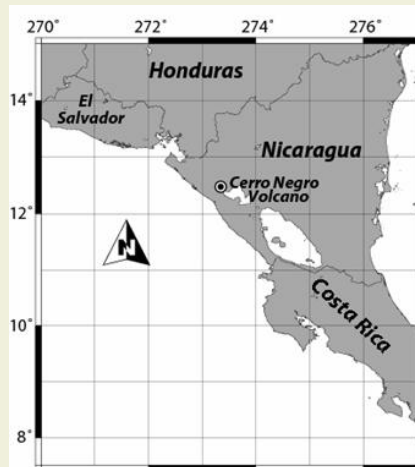
(3) Perhaps the best method involves using a computer program to model the eruption itself, finding the best-fit to the observed tephra thickness values using a numerical simulation of the eruption. This method is highly effective and does not require interpolation of isopachs, but does require a great deal of computation.



# Background

*Cerro Negro volcano is a cinder cone located in the Central American volcanic arc, created by the subduction of the Cocos plate beneath the Caribbean plate. Cerro Negro first formed in 1850 and has experienced more than 20 eruptions since its formation. One of the largest of these eruptions occurred in April, 1992. With little warning, a sub-Plinian eruption column developed that reached about 7 km height. When the 2 day eruption was over, 20,000 people had been evacuated, mostly due to failed water systems. Crop damage was widespread and catastrophic, and some roofs had collapsed in the nearby city of Leon, the second largest city in Nicaragua.*

## Location of Cerro Negro volcano



## The 1992 Eruption of Cerro Negro



Photo by C. Connor

*Building collapse as a result of tephra fallout during the 1992 eruption of Cerro Negro. In this area, just east of the city of Leon, tephra fallout accumulation was only about 4 cm, but sufficient to cause collapse of this poorly constructed roof.*

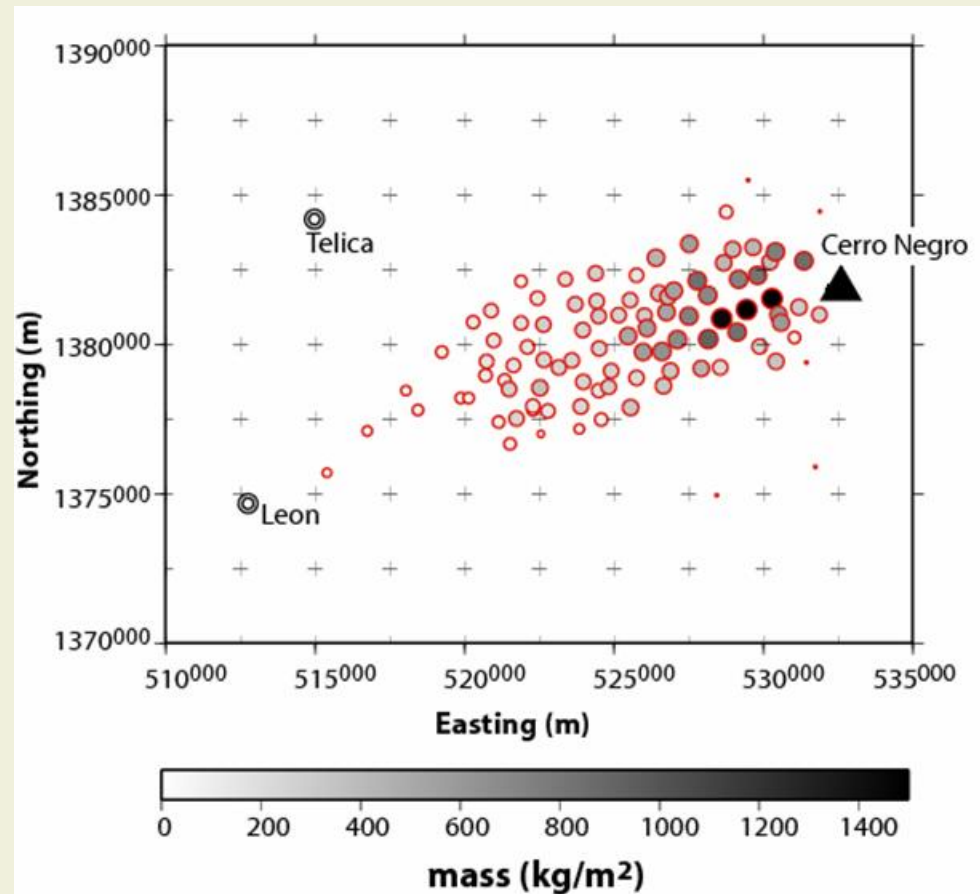
[Learn more about the consequences of the eruption](#)

# Problem

What is the volume of the 1992 tephra fallout deposit?

*Data are required to estimate the volume of the tephra fallout deposit. In this case, the data comes from pits dug in the tephra fallout deposit. These pits are used to record thickness of the deposit, density of the deposit, and grain size. On the map, tephra fallout is represented as mass per unit area.*

*Given a deposit thickness and deposit density – make sure you can calculate the mass per unit area – a value much more useful than thickness for understanding potential damage.*



Connor and Connor, 2006

[Learn more about how this map was made](#)

# Designing a Plan, Part 1

Given information about the area enclosed by specific isopach lines, estimate the volume of the 1992 eruption.

You will need to:

- Plot the natural logarithm of isopach thickness (m) against the square root of isopach area (km).
- Find a best-fit linear model for these data.
- Estimate parameters (slope and y-intercept) from your best-fit linear model.
- Estimate deposit volume from these parameters.

Give answer in cubic kilometers.

*Notes:*

The model you will use is based on the exponential-thinning model. That is, it is assumed that the thickness of the deposit, on average, decays exponentially away from the vent of the volcano (hence  $\ln(\text{Thickness})$  is plotted).

Square-root of isopach area is used in place of distance from the volcano to average out the effects of the wind.

You will use the excel “trend line” function to fit a line to the data and to estimate the parameters of the model.

With these parameters in hand, you will estimate the volume of the deposit.



## Designing a Plan, Part 2

Use the exponential-thinning model of Pyle (1989) to estimate the volume.

In the exponential-thinning model, deposit thickness,  $T$ , depends on area of the deposit,  $A$ , that has thickness greater than  $T$ , an initial thickness at the vent,  $T_o$ , and a thinning constant,  $k$ :

$$T = T_o \exp\left[-kA^{1/2}\right]$$

The equivalent half-thickness is the average distance from the vent at which the deposit reaches one half its initial thickness. This is a measure of dispersion of the deposit. Think of  $b_T$  as equivalent to the half-life in radioactive decay.

$$b_T = \frac{\ln(2)}{k\sqrt{\pi}}$$

The volume of the deposit is then estimated using the parameters  $T_o$  and  $b_T$ :

$$V = 13.08T_o b_T^2$$

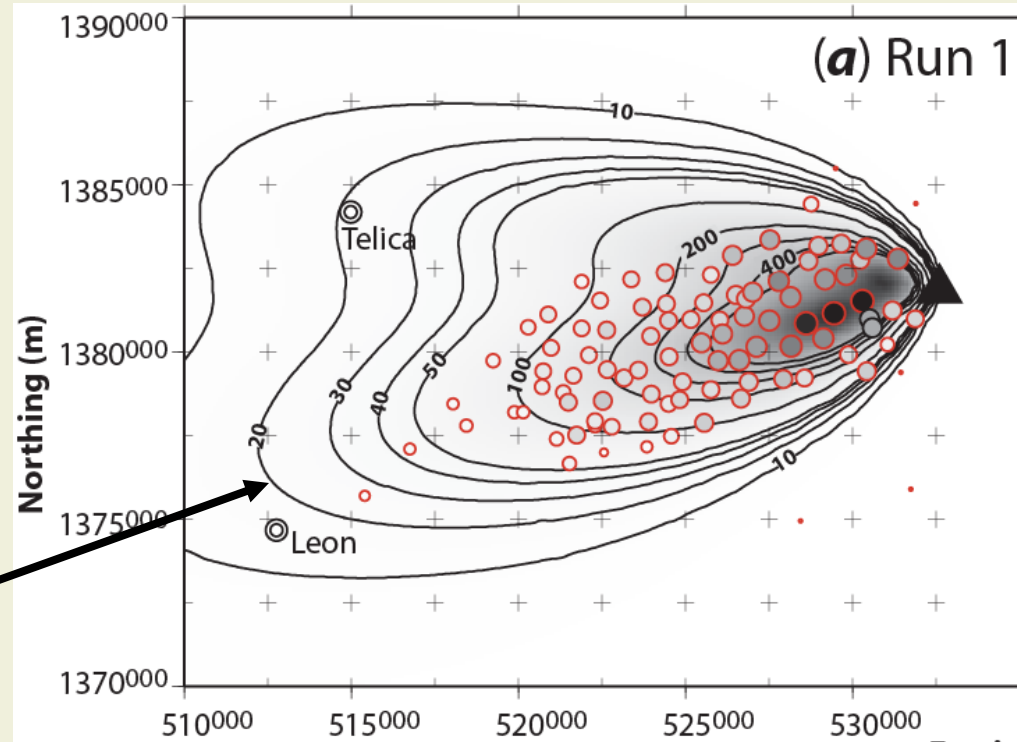
[Learn about where these equations come from](#)

# Carrying out the Plan, Part 1: Isopach area

The first step is to convert information on the isopach (or isomass) map to a form used in the equations to calculate volume. In this case, you are given the area enclosed by isopachs of a given thickness. For example, the 2 cm isopach encloses approximately 181 km<sup>2</sup> area.

*Volume is determined by estimating the parameters  $T_0$  and  $b_T$  using the equations of the previous slide.*

	B	C	D	E
2	<b>Calculate the Volume of the Tephra Deposit</b>			
3				
4	<b>Given</b>			
5	Isopach Area	Thickness		
6	(km <sup>2</sup> )	(m)		
7	16.25	0.4		
8	30.63	0.2		
9	58.87	0.1		
10	95.75	0.05		
11	181.56	0.02		
12	275.10	0.01		



Isomass map (kg m<sup>-2</sup>) for the 1992 eruption  
From Connor and Connor, 2006.

*20 kg m<sup>-2</sup> is equivalent to 2 cm thickness, if the density of the tephra fallout deposit is 1000 kg m<sup>-3</sup>. Please be sure you understand this conversion.*

# Carrying out the Plan, Part 2: Calculations and Plotting the data

Update your spreadsheet to calculate the  $A^{1/2}$  and  $\ln(T)$  ; then plot the graph.

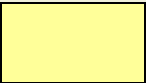
Note that:

$$T = T_o \exp \left[ -kA^{1/2} \right]$$

can be simplified as:

$$\ln T = \ln T_o - kA^{1/2}$$

a linear equation. So the slope,  $k$ , and the intercept  $\ln(T_o)$  may be estimated from a plot of  $\ln(T)$  vs  $A^{1/2}$ .

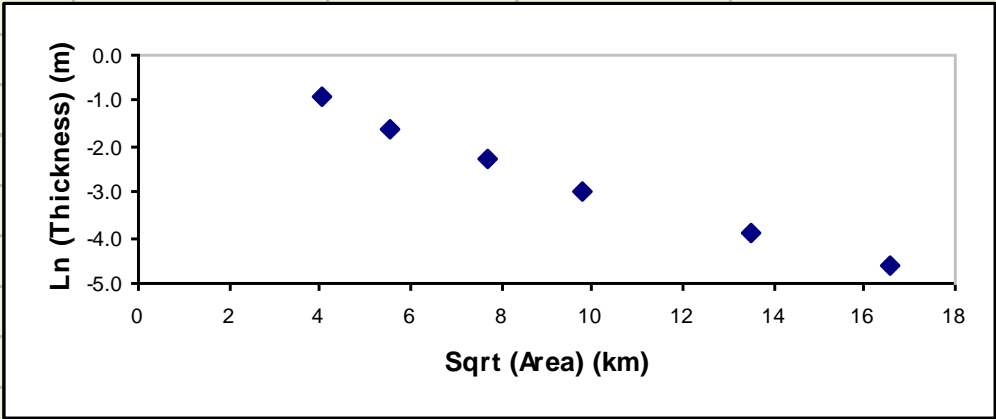


A cell containing given information



A cell containing a formula

	B	C	D	E
2	<b>Calculate the Volume of the Tephra Deposit</b>			
3				
4	<b>Given</b>		<b>Calculated</b>	
5	Isopach Area	Thickness	Sqrt (Area)	Ln (Thickness)
6	(km <sup>2</sup> )	(m)	(km)	Ln (m)
7	16.25	0.4	4.031129	-0.91629
8	30.63	0.2	5.534438	-1.60944
9	58.87	0.1	7.672679	-2.30259
10	95.75	0.05	9.785193	-2.99573
11	181.56	0.02	13.474420	-3.91202
12	275.10	0.01	16.586139	-4.60517



# Carrying out the Plan, Part 3: Finding the Model Parameters

The parameters  $T_0$  and  $b_T$  are estimated by fitting a linear equation to the data.

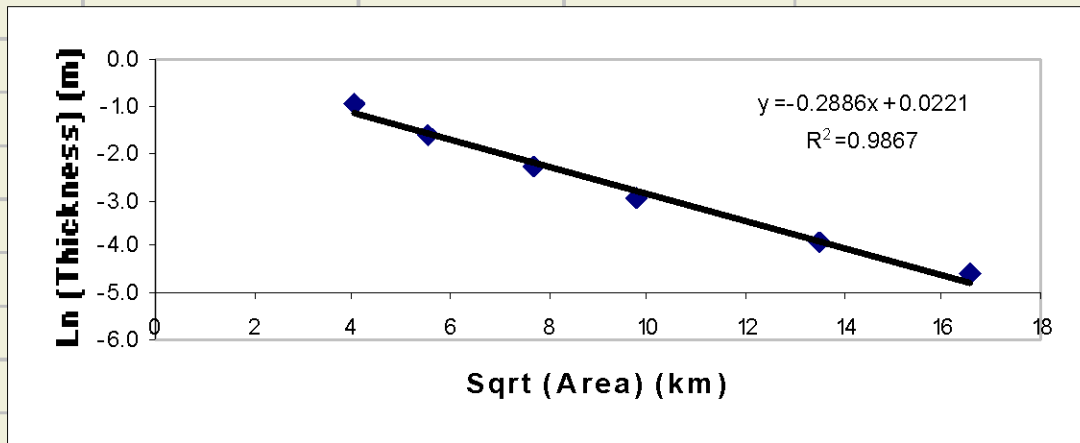
**Use your Excel spreadsheet to fit a “trendline” and estimate the parameters. Make sure you see that**

$$k = 0.289$$

and

$$\ln(T_0) = 0.022$$

	B	C	D	E
2	<b>Calculate the Volume of the Tephra Deposit</b>			
3				
4	<b>Given</b>		<b>Calculated</b>	
5	Isopach Area	Thickness	Sqrt (Area)	Ln (Thickness)
6	(km <sup>2</sup> )	(m)	(km)	Ln (m)
7	16.25	0.4	4.031129	-0.91629
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12	275.10	0.01	16.586139	-4.60517



# Carrying Out the Plan, 4: Spreadsheet to Calculate the Volume

Once  $k$  and  $\ln(T_o)$  are estimated, the values of  $b_T$  and volume are determined:

$$b_T = \frac{\ln(2)}{k\sqrt{\pi}}$$

$$V = 13.08T_o b_T^2$$

	B	C	D	E
2	<b>Calculate the Volume of the Tephra Deposit</b>			
3				
4	<b>Given</b>		<b>Calculated</b>	
5	Isopach Area	Thickness	Sqrt (Area)	Ln (Thickness)
6	(km <sup>2</sup> )	(m)	(km)	Ln (m)
7	16.25	0.4	4.031129	-0.91629
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9	58.87	0.1	7.672679	-2.30259
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11	181.56	0.02	13.474420	-3.91202
12	275.10	0.01	16.586139	-4.60517
13				
14	k	0.289		
15	ln(T <sub>o</sub> )	0.022		
16	b <sub>T</sub>	1.353171	km	
17	T <sub>o</sub>	1.022244	m	
18	V	0.024483	km <sup>3</sup>	

**Modify your spreadsheet to perform the final tasks in your volume calculation.**

**Be sure you keep careful track of units in this calculation.**

**Be sure you can easily change the units of volume from km<sup>3</sup> to m<sup>3</sup>.**

**Verify that the 1992 eruption of Cerro Negro was VEI 3.**

These parameters are estimated by linear regression, using the “trend line” function in Excel.



# What you have done

You have calculated the volume of a tephra deposit given data about the area enclosed by specific isopachs of deposit thickness, derived from observations of the deposit. With this information, you have categorized the 1992 Cerro Negro eruption in terms of the Volcano Explosivity Index.

An important lesson is that volume estimation of geologic deposits can involve the application of a model. In this case, a statistical model (exponential) is used to estimate the volume of the deposits from observations. Hopefully, the application of a model, rather than simply summing the areas of isopachs, improves the estimate of the volume.

Of course, there are several sources of uncertainty in this estimate. First, as you have applied the exponential model here, you must rely on the interpolation (even extrapolation!) of isopach lines and the estimation of areas enclosed by these lines. Second, the exponential model may not apply well to the data. In fact, for some deposits of  $\ln(T)$  vs.  $\sqrt{\text{Area}}$  the pattern is decidedly nonlinear.

Whenever a model is used, it is important to (1) understand the underlying assumptions, (2) apply the model correctly, (3) evaluate the result of your analysis, both in terms of data quality and model performance.

*There is much more to estimation of the volume of tephra fallout deposits. See:*

Pyle, D., 1989, The thickness, volume, and grainsize of tephrafall deposits, *Bulletin of Volcanology*, 51: 1-15.

Fierstein, J., and M. Nathenson, 1993, Another look at the calculation of tephra fallout volumes, *Bulletin of Volcanology*, 56: 121-132.

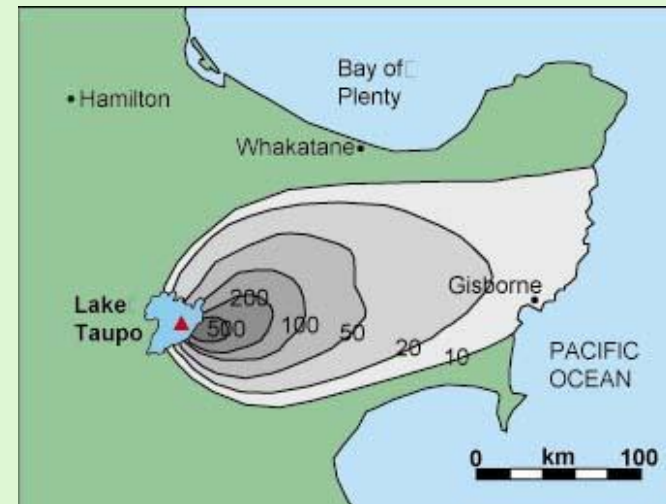
Bonadonna, C., and B. F. Houghton, 2005, Total grainsize distribution and volume of tephra fallout deposits, *Bulletin of Volcanology* 67: 441-456.

Connor, L.J., and C.B. Connor, 2006, Inversion is the solution to dispersion: understanding eruption dynamics by inverting tephra fallout, In: Mader, H.M., Coles, S.G., Connor, C.B., and Connor, L.J. (editors) *Statistics in Volcanology*, Special Publications of IAVCEI 1, Geological Society, London, 231-242.

# End of Module Assignments

1. Make sure you turn in a spreadsheet showing your estimated volume of the 1992 Cerro Negro eruption.
2. Given your volume estimate, assume the density of the tephra deposit is  $1000 \text{ kg m}^{-3}$ , and the density of the melt that actually erupted was  $2800 \text{ kg m}^{-3}$ . What was the volume of melt (as opposed to tephra) erupted? This latter volume is often referred to as the dense rock equivalent (DRE) volume.
3. Given the following table of information, use your spreadsheet to make volume estimates for the 1902 eruption of Santa Maria (Guatemala) and the AD 186 Taupo eruption (New Zealand). Estimate the VEI of these eruptions.

Santa Maria		Taupo	
Area (km <sup>2</sup> )	Thickness (cm)	Area (km <sup>2</sup> )	Thickness (cm)
255	150	441	138
1483	100	1369	97
5932	50	3025	66
27699	10	8100	30
86642	1	14400	15

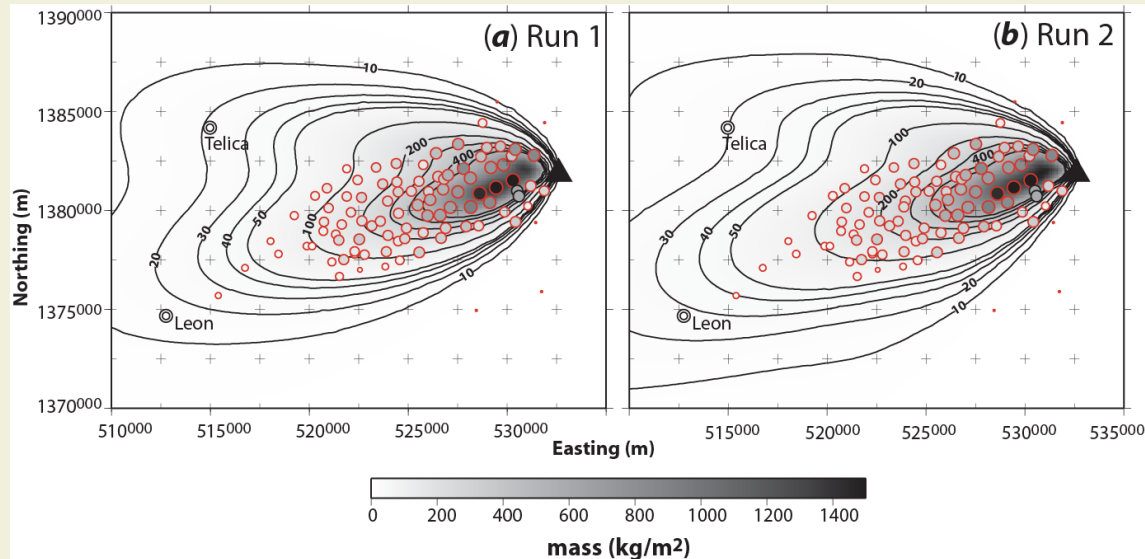


Taupo isopach (Wilson and Walker 1985 from [http://volcano.und.edu/vwdocs/volc\\_images/australia/new\\_zealand/taupo.html](http://volcano.und.edu/vwdocs/volc_images/australia/new_zealand/taupo.html))

4. Given the information about the 2450 BP eruption of Pululahua volcano (Ecuador) on slides [21](#) and [22](#): (a) contour the thickness values to create an isopach map, (b) assuming circular isopachs, calculate the area enclosed by a set of at least 5 isopachs, (c) estimate the volume of the deposit (Please thank Alain Volentik for data used in this problem!)

# Computer Models of Tephra Dispersion

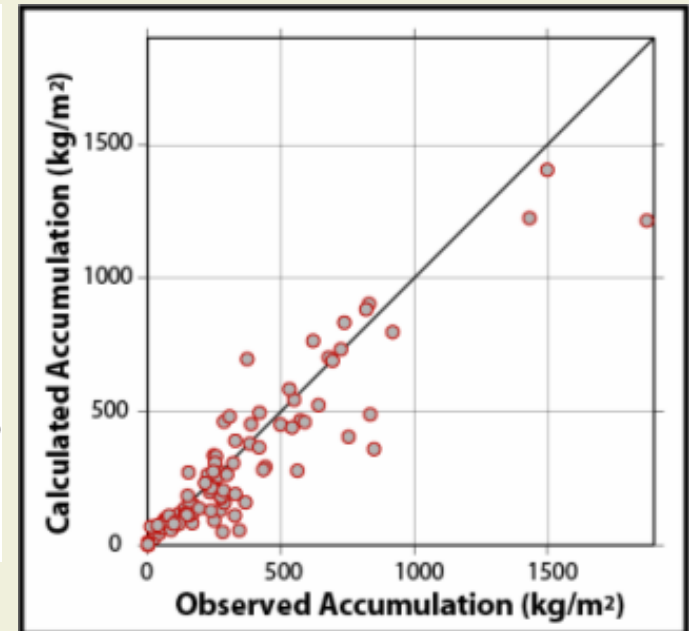
A problem with making isopach maps is that usually a great deal of interpolation is required. Most maps are made by contouring few thickness values! An alternative approach to volume estimation is to use a numerical simulation of the eruption. The “forward model” estimates the accumulation of tephra, given specifics about the eruption, such as the volume, column height, and grain size distribution. An “inverse model” finds the best-fit between observed thickness (or mass per unit area) and calculated values, using the forward model. These simulation results show the inverse solution for Cerro Negro – 1992. The estimated volume is about  $3 \times 10^7 \text{ m}^3$ , close to the value you will find by the exponential-thinning model.



These two runs show reasonable, best-fit models and the range of eruption parameters used to make the isomass maps.

From Connor and Connor, 2006.

Best-Fit Parameters	calculated-Run 1	calculated-Run 2	observed	(units)
Maximum Column Height	6010	6406	7000-7500	(m)
Total Mass Ejected	2.99e10	3.62e10	2.35e10	(kg)
Mean Particle Size	0.03	0.0	-0.1	(phi)
Std. Dev. of particle diameter	1.18	1.98	1.4	(phi)



This graph shows the goodness-of-fit of the model for Run 1.

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# More about the Consequences of the 1992 Eruption

Although the 1992 Eruption of Cerro Negro was comparatively small, it had dire impacts on the community living about the volcano. Approximately 20,000 people were displaced by the eruption (although estimates vary widely). The photo at left shows children in a Red Cross evacuation center set up in harsh conditions. The waning stage of the eruption is seen in the background. Crop failure was a major consequence of the eruption, an extreme hardship for people living at a near subsistence level on the farm land around the volcano. The school house (below), was saved when members of the community removed the roof during tephra fallout, avoiding building collapse and much greater damage.





# How was the Map Made?

Isopach maps are made by sampling the deposit in as many locations as possible. At each sample site the deposit is trenched to expose the entire thickness of the tephra fallout layer.

At Cerro Negro, the 1992 deposit is easily recognized by abrupt changes in grain size and color (deuteric alteration of scoria derived from the older cone are present throughout the lower layer of the 1992 deposit.) The isopach lines are interpolated from these sample locations.

Then the deposit is sampled for granulometric studies, its density measured, and stratigraphic features noted.



*Photos by C. Connor*



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# Calculating the Volume of an Exponentially Thinning Deposit

David Pyle (1989, The thickness, volume, and grainsize of tephra fall deposits, Bulletin of Volcanology 51: 1-15) developed a method to estimate the volume of an exponentially thinning deposit that is still in wide use today. The method assumes: exponential thinning of the deposit with distance, and elliptical isopach with the source at one focus of the ellipse.

In the exponential-thinning model, deposit thickness,  $z$ , depends on distance from the vent,  $r$ , an initial thickness at the vent,  $T_o$ , and a thinning constant,  $k$ :

$$z = T_o \exp[-kr]$$

$$dz = kT_o \exp(-kr)dr$$

$$dV = xy\pi dz$$

$$\frac{y}{x} = \alpha = (1 - e^2)^{1/2}$$

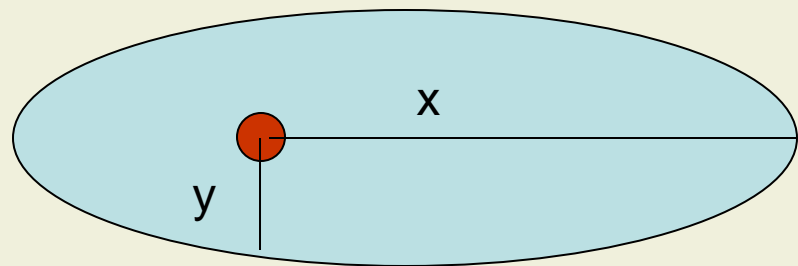
$$dV = \frac{(\pi y^2 dz)}{\alpha}$$

$$V = \frac{1}{\alpha} \int_0^{\infty} \pi r^2 k T_o \exp(-kr) dr$$

$$V = \frac{2\pi T_o}{\alpha k^2} = \frac{2\pi T_o b_T^2}{\alpha (\ln 2)^2} = 13.08 T_o b_T^2 / \alpha$$

$$V = 13.08 T_o b_T^2 \quad \text{For } \alpha = 1$$

Ellipse with eccentricity  $\alpha$  and the volcano at one focus



# Doing a Linear Regression in Excel

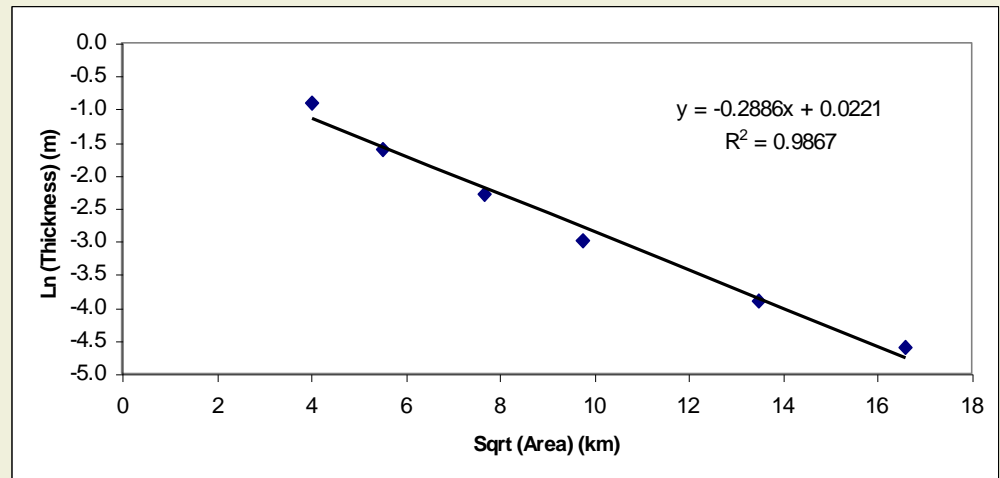
Learn more about the [mathematical basis of linear regression](#) and [correlation coefficients](#)

Steps in doing a linear regression in Excel:

0. Be sure you are on the worksheet which contains the chart you wish to work with.
1. Move the mouse cursor to *any data point* and press the left mouse button. All of the data points should now be highlighted. Now, while the mouse cursor is still on any one of the highlighted data points, press the right mouse button, and click on **Add Trendline** from the menu that appears.
2. From within the "Trendline" window, click on the box with the type of fit you want (e.g., Linear).
3. Click on **Options** at the top of the "Trendline" window.
4. Click in the checkbox next to "Display Equation on Chart" and the checkbox next to "Display R-squared Value on Chart". **Do not click on the checkbox next to "Set Intercept = 0"**.
5. Click **OK**. A line, an equation, and an R-squared value should appear on the graph.

$R^2$  is a measure of goodness-of-fit of the linear regression model.

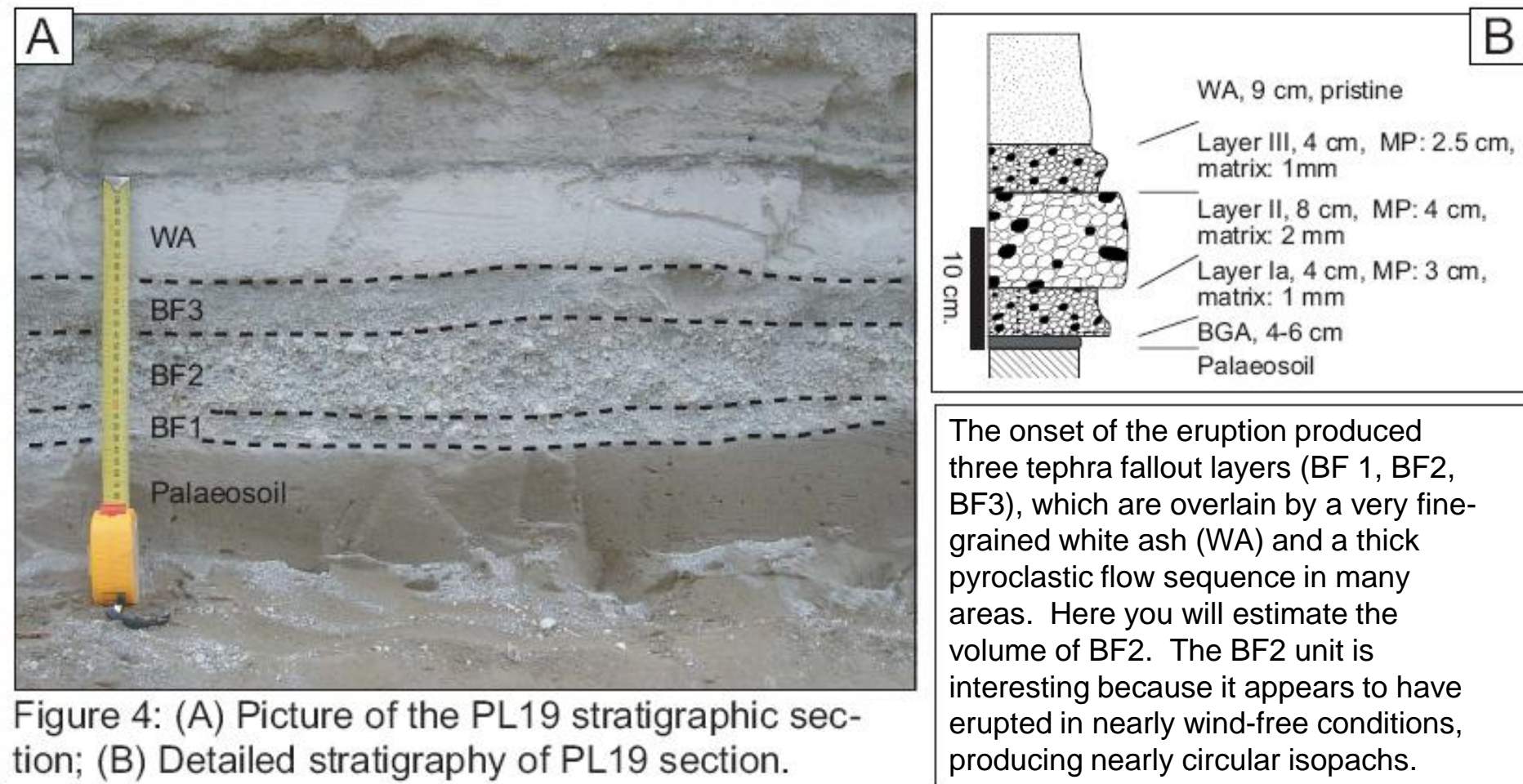
**Make sure you have a good understanding of the basics of linear regression and a qualitative understanding of how to interpret correlation coefficients.**



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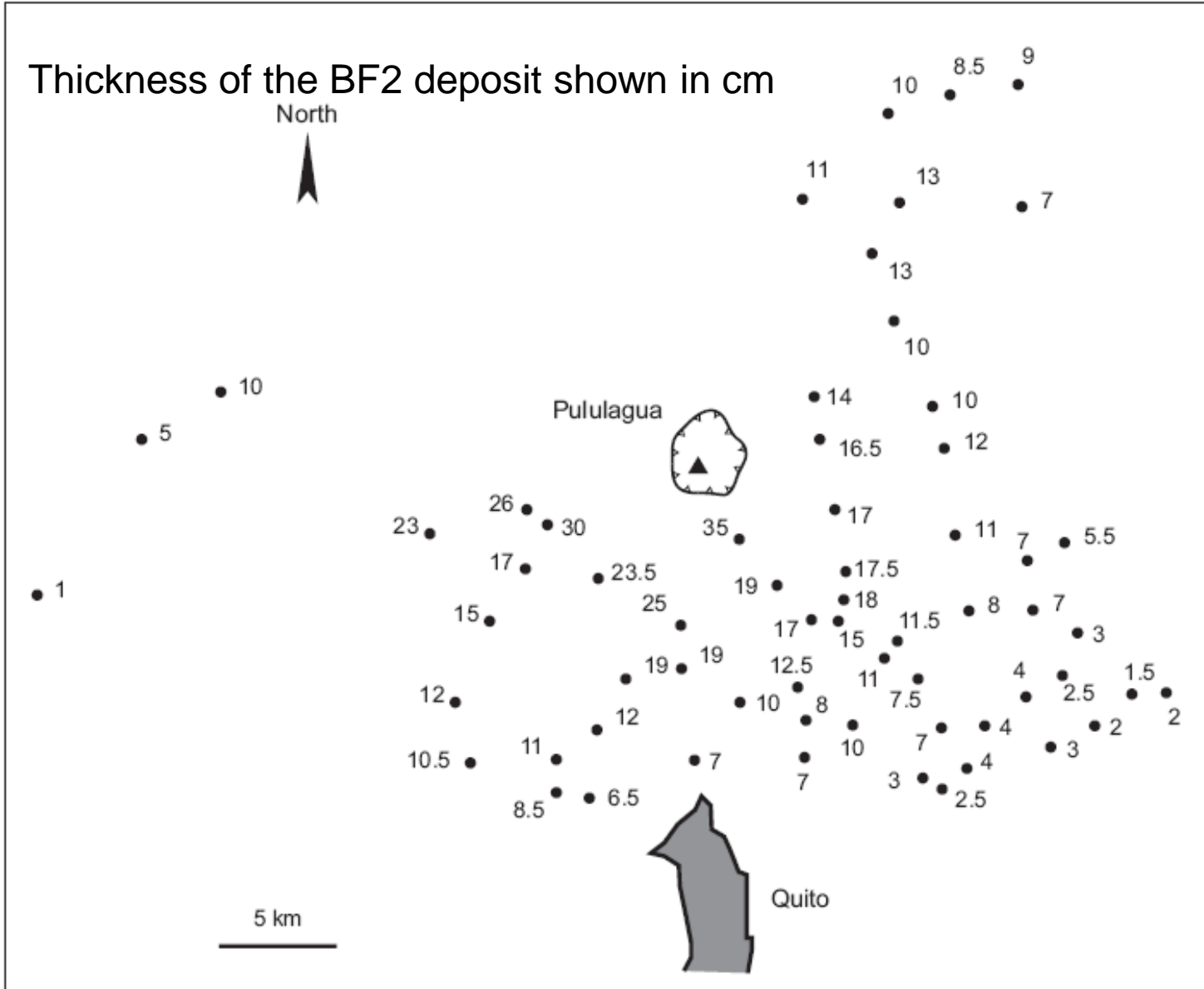
# EOM Assignment Question 4: the Pululahua Deposit

The 2450 BP eruption of Pululahua volcano, Ecuador, formed a caldera just north of the present capitol city of Ecuador – Quito. This eruption likely caused widespread death and destruction in the Inter-Andean Valley.



The onset of the eruption produced three tephra fallout layers (BF 1, BF2, BF3), which are overlain by a very fine-grained white ash (WA) and a thick pyroclastic flow sequence in many areas. Here you will estimate the volume of BF2. The BF2 unit is interesting because it appears to have erupted in nearly wind-free conditions, producing nearly circular isopachs.

# EOM assignment question 4: The deposit thickness map



Print out this map and contour the thickness values.

Be sure to show at least 5 isopach lines.

Be sure to dash the isopach lines where you are forced to interpolate a great deal.

Be sure to label the isopach lines.

Use this map to help you estimate the area enclosed by each isopach line. (*big hint*: for purposes of the calculation, assume the isopachs are circular!).